Exp d'une matrice

Def $\longrightarrow u \in \mathcal{L}(E)$, $\exp(u) = \sum_{n=0}^{+\infty} \frac{u^n}{n!}$ $\left(|||u||| \leqslant R = +\infty \right)$

$u \mapsto \exp u$ $\mathcal{C}^0$

$\longrightarrow [u, v] = 0 \implies e^{u+v} = e^u e^v$ (preuve : Majorer la diff avec $|||\cdot|||$ puis CV en réels)

$\longrightarrow f : B_{|||\cdot|||}(0, R) \longrightarrow \mathcal{L}(E)$, $u \longmapsto \sum_{n=0}^{+\infty} a_n u^n$ est $\mathcal{C}^0$ (R rayon de CV)

$\longrightarrow |||u||| \cancel{\leqslant} R$ (R de CV) $\implies \sum a_n u^n$ CV

$\longrightarrow \exp A \in \mathbb{C}[A]$ ($\mathbb{R}$ aussi...)

$\longrightarrow A$ DZ $\overset{\text{Exo classique}}{\Longleftrightarrow} e^A$ DZ

$\longrightarrow \exp(\mathcal{M}_d(\mathbb{C})) = GL_d(\mathbb{C})$

- $\triangleright$ $A \in \mathcal{M}_n(\mathbb{R}) \to \det e^A > 0$
- $\triangleright$ $\operatorname{spec} A = (\lambda_1, \dots, \lambda_n) \to \operatorname{spec} e^A = (e^{\lambda_1}, \dots, e^{\lambda_n})$
- $\triangleright$ $\det e^A = e^{\operatorname{Tr} A}$